Anomalous optical transmission through a vortex lattice in a film of type-II superconductor

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We study the effect of anomalous optical transmission through an array of vortices in a type-II superconducting film subjected to a strong magnetic field. The mechanism responsible for this effect is resonance transmission between two surface plasmon polaritons (SPP) in the system. The SPP band gap in the system is studied as a function of magnetic field and temperature. Control of transmission by varying magnetic field and/or temperature is analyzed.

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I. INTRODUCTION

Extraordinary optical transmission through arrays of holes of subwavelength diameter in metal films has been the subject of extensive study since detection of large enhancements in transmitted intensity was first reported.[1] Several mechanisms responsible for this enhancement have been discussed, including excitation of surface plasmon polaritons (SPPs) of the film surfaces.[2,3] In this Paper we propose a novel physical realization for studying possible anomalous optical transmission through a thin film. In particular, we consider a film comprised of a type-II superconductor subjected to a strong magnetic field, which causes a phase transition of the superconductor to a state with a lattice of Abrikosov vortices.[4,5] Due to the dielectric contrast between regions inside Abrikosov vortices and outside them, these vortices can behave in a manner analogous to an array of holes in the metal. By controlling the properties of the Abrikosov lattice of vortices via external magnetic field strength and temperature, it is possible to change the light transmission through the lattice of vortices. We consider resonant light transmission by the vortices and calculate the properties of SPPs excited in the film by the incident radiation. The incident light falls on the front surface of the superconducting film. We consider the following scenario of anomalous light transmission through the film (analogously to Ref. [6] and see also the references cited therein). The incident light excites an SPP on the front surface. Subsequently, the front SPP resonantly excites through the vortices an SPP on the opposite surface of the film. Finally, this second SPP emits photons from the opposite side of the film.

Photonic crystals[7,8,9,10] based on superconductors have been the focus of several recent works.[11,12,13,14]. The optical properties of superconductors have been extensively studied (see, for example, Refs. [4,5,15] and references therein). There are two critical magnetic fields for a type-II superconductor: B_{c1} and B_{c2}. When the external magnetic field B is smaller than B_{c1} light does not penetrate a distance deeper than a characteristic skin depth λ. When $B_{c1} < B < B_{c2}$ there is a mixed state created by Abrikosov vortices of normal metallic phase in a superconducting medium. The Abrikosov vortices arrange into a two-dimensional (2D) triangular lattice. There is dielectric contrast between the Abrikosov vortices of normal metallic phase and the surrounding superconducting medium resulting in the appearance of a photonic band gap. At $B > B_{c2}$ the material has the properties of the normal metal.

In recent experiments, superconducting (SC) metals (Nb in particular) have been used as components in optical transmission nano-materials.[16] It was found that dielectric losses are substantially reduced in the SC metals relative to analogous structures made out of normal metals, and also that band edges tend to be sharper with the SC metals.

Here we study optical transmission through a film of thickness h comprised of a type-II superconductor subjected to an external perpendicular magnetic field such that the superconductor is in the mixed state $B_{c1} < B < B_{c2}$. We consider a highly anisotropic superconductor where the axis corresponding to large plasma frequency is perpendicular to the superconducting film, and the axis corresponding to small plasma frequency is located in the plane parallel to the film. An example of such a superconductor is a quasi-one-dimensional type-II superconductor.[17,18] We will choose the incident light direction so that light propagates along the axis of this quasi-one-dimensional superconductor. The corresponding electric field excites the polarization in the direction perpendicular to the axis. The plasma frequency in the direction perpendicular to the axis is sufficiently small that the photonic band gap is located within the region of the superconducting gap. We show herein that the surface plasmon polariton (SPP) band gap and hence light transmission[6,19] are tunable via external magnetic field B and temperature T. This paper is organized as follows. In Sec. II we calculate the SPP band gap for an SC film. In Sec. III we describe and discuss our results concerning a tunable SPP crystal in a type-II SC film subjected to an appropriate external magnetic field.

II. SURFACE PLASMON POLARITON (SPP) BAND GAP

We will consider a system of Abrikosov vortices in a type-II superconductor that are arranged in a triangular lattice. The axes of the vortices (directed along the $\hat{z}$ axis) are perpendicular to the surface of the superconductor. We assume the $\hat{x}$ and $\hat{y}$ axes to be parallel to the two real-space lattice vectors that characterize the 2D triangular lattice of Abrikosov vortices in the film (the angle between $\hat{x}$ and $\hat{y}$ is $\pi/3$). The nodes of the 2D triangular lattice of Abrikosov vortices are assumed to be placed on the x and y axes. We assume that the superconducting film is surrounded by two types of media at the two interfaces characterized by dielectric constants ε_I and ε_{III}, respectively. The dielectric constant of the superconducting film is determined by the magnetic field $\varepsilon_{II} = \varepsilon_{eff}$; the specific functional form of ε_{eff} is discussed below.

For simplicity, we consider the superconductor in the London approximation[4] (i.e., assuming that the London penetration depth λ_0 of the bulk superconductor is much greater than the coherence length ξ; here $\lambda_0 = [m_e c^2/(4\pi n_e e^2)]^{1/2} \gg \xi$; $\xi \sim \hbar v_F \Delta^{-1}$; n_e is electron density; m_e and e are the mass and the charge of the electron, respectively; Δ is the superconducting gap; v_F is the Fermi velocity). Note that London penetration depth of the thin ($h \lesssim \xi$) superconducting film is λ_0^2/h, where h is the thickness of the film.

Abrikosov vortices of radius ξ arrange themselves into a 2D triangular lattice with lattice spacing $a(B,T)$ at fixed magnetic field B and temperature T:[13]

$$a(B,T) = 2\xi(T)\sqrt{\frac{\pi B_{c2}}{\sqrt{3}B}}. \tag{1}$$

Thus the period of the vortices in the 2D lattice depends on the temperature of the superconductor and the applied magnetic field strength.

Although light itself cannot penetrate into the film further than the skin-depth, it can excite surface-plasmon polaritons (SPPs), which are collective excitations of the electron plasma at a surface of metal (surface plasmons) coupled with photons.[6,19] These SPPs can be resonantly coupled through the vortices. The dielectric constant in medium II $\varepsilon_{II}(x,y) = \varepsilon_{eff}(x,y,\omega)$ is taken to be periodic in the x and y directions:

$$\varepsilon_{II}(x,y) = \varepsilon_{II}(x+na, y+ma), \tag{2}$$

where n and m are integers.

We employ the following Fourier expansions for the dielectric constant and electric field:

$$\begin{aligned} \varepsilon_{II} &= \sum_{n=-\infty}^{\infty} e_n \exp(ingx), \\ \vec{E} &= \sum_{n=-\infty}^{\infty} \vec{E}_n \exp(ingx), \end{aligned} \tag{3}$$

where $g(B,T) = 4\pi/(\sqrt{3}a(B,T))$ is the reciprocal lattice vector directed along the x axis connecting the nearest nodes of the 2D triangular lattice of the Abrikosov vortices. We have omitted the expressions for the y components of the Fourier expansions since they are analogous to the expressions for the x components, and we consider only SPPs propagating in the x direction. Using a weak field treatment (due to the small dielectric contrast inside vs. outside the vortices), and retaining only the two first components of the Fourier series, we have[6]

$$\varepsilon_{II}(x) = \varepsilon_0 + \varepsilon_1 \cos(gx). \tag{4}$$

This approximation is valid when the distance between two nearest Abrikosov vortices is not much greater than the size of an Abrikosov vortex (the distance between two nearest Abrikosov vortices is about the same order of magnitude as the size of an Abrikosov vortex). This condition is true at temperatures close to T_c, which is the situation of direct interest in this work.

For band-gap calculations in a three-wave approximation[6] the electric field in the film can be represented as

$$\vec{E} = \left[\vec{A} + \vec{B}\cos(gx) + \vec{C}\sin(gx)\right] e^{\kappa z}. \tag{5}$$

Applying Maxwell equations

$$\begin{aligned} \left(\varepsilon_{II}k_0^2 + \frac{\partial^2}{\partial z^2}\right) E_x - \frac{\partial^2 E_z}{\partial z \partial x} &= 0, \\ \left(\varepsilon_{II}k_0^2 + \frac{\partial^2}{\partial x^2}\right) E_z - \frac{\partial^2 E_x}{\partial z \partial x} &= 0, \end{aligned} \tag{6}$$

where $k_0 = \omega/c$, a set of six equations for the six amplitudes A_x, A_z, B_x, B_z, C_x, and C_z can be easily obtained.[6] This set of six equations splits into two independent sets of three equations for the fields A_x, B_x, C_z and A_z, B_z, C_x, respectively. The conditions of solvability for each of these two independent sets of three equations are exactly the same as for the other set, and they result in the following expressions for three eigenvalues κ:

$$\begin{aligned}\kappa_{1,2}^2 &= \frac{1}{2}\left[g^2 - 2\varepsilon_0 k_0^2 \mp \sqrt{g^4 + 8\alpha_1\varepsilon_0 k_0^2(\varepsilon_0 k_0^2 - g^2)}\right], \\ \kappa_3^2 &= -\varepsilon_0 k_0^2 + \frac{g^2}{1-2\alpha_1}, \end{aligned} \tag{7}$$

where $\alpha_1 = \varepsilon_1^2/\varepsilon_0^2$ is assumed to be small, i.e., $\alpha_1 \ll 1$. To derive the SPP dispersion law and relations between the constructed fields, one needs to satisfy appropriate boundary conditions at the interface, namely, continuity of: (i) the x component of the electric field and (ii) the z component of the electric displacement vector $D_z = \varepsilon E_z$ (cf. Eqs. (14) and (15) of Ref. [6]). In the case of periodicity in the surface that arises due to Abrikosov vortices, a band gap opens around a particular frequency value $\omega_0(g)$ bracketed by lower frequency ω_a and upper frequency ω_b. The following expressions for the frequencies of the band-gap edges can be obtained:

$$\omega_a \approx \omega_0(1 + \Delta_1 + i\Delta_2), \tag{8}$$

where

$$\Delta_1 \approx \frac{-\alpha_{01}[\beta^3 + 2\beta^2 + 3\beta + 2 - 2(\beta^2+\beta+1)\sqrt{1+\beta}]}{(1-\beta^2)(1-\beta)\beta} < 0, \tag{9}$$

$$\Delta_2 \approx \frac{2\alpha_{01}[2 + 2\beta - (2+\beta)\sqrt{1+\beta}]}{(1-\beta^2)(1-\beta)\sqrt{-\beta}} < 0, \tag{10}$$

$$\omega_0^2 = c^2 g^2 \frac{\varepsilon_I(\omega_0) + \varepsilon_0(\omega_0)}{\varepsilon_I(\omega_0)\varepsilon_0(\omega_0)}, \tag{11}$$

$$\omega_b \approx \omega_0\left[1 - \frac{\alpha_{01}\beta(2+\beta)}{1-\beta^2}\right], \tag{12}$$

with $\alpha_{01} = \alpha_1(\omega = \omega_0)$ and $\beta = \varepsilon_I/\varepsilon_0(\omega = \omega_0)$. The following inequality holds: $Re(\omega_a) < Re(\omega_0) < Re(\omega_b)$. Therefore, a forbidden gap $(Re(\omega_b) - Re(\omega_a))$ is opened up in the spectrum of the SPPs. We assume here that the imaginary parts of these frequencies, which lead to damping of the excitations, are negligible.

We shall utilize the framework of the two-fluid model to describe the dielectric function inside and outside the vortex. In this model we assume that inside every vortex there exists a normal metal state whose dielectric function can be described via a simple Drude model:

$$\varepsilon_{in}(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}, \tag{13}$$

where $\omega_p = \sqrt{4\pi n_e e^2/m_e}$ is the plasma frequency of the normal metal, and γ accounts for damping in the normal conducting state.

Outside the vortices there are both normal and superconducting components of the metal state. The density of each component (n_n, n_s) depends on temperature. Near the critical temperature we can write for n_n, n_s that $n_s/n_n \simeq 2(T_C - T)/T_C$. Then the dielectric function takes the form:

$$\varepsilon_{out}(\omega) = 1 - \frac{\omega_{ps}^2}{\omega^2} - \frac{\omega_{pn}^2}{\omega(\omega + i\gamma)}, \tag{14}$$

where $\omega_{ps}^2 = 4\pi n_s e^2/m_e = 2\omega_p^2(T_C - T)/T$ and $\omega_{pn}^2 = 4\pi n_n e^2/m_e = 4\pi(n_e - n_s)e^2/m_e = \omega_p^2(2T - T_C)/T$.

The weak field model for the dielectric function of the superconducting film $\varepsilon_{II}(x)$ given by Eq. (4) is valid for small dielectric contrast inside and outside of the vortices, which is the case at temperatures T close to T_c. The contrast of

imaginary parts of dielectric constants inside and outside of a vortex, α_{Im}, does not depend on frequency and can be written as follows:

$$\alpha_{Im} = \frac{Im(\varepsilon_{out}) - Im(\varepsilon_{in})}{Im(\varepsilon_{out}) + Im(\varepsilon_{in})} = (T_C - T)/T \to 0, T \to T_C \tag{15}$$

At the temperature $(T_C - T)/T_C = 0.022$, for example, α_{Im} is about 2%, and we can reduce it further by increasing the temperature closer to T_C. Hence we will neglect it in what follows. The contrast between the real parts of dielectric constants inside and outside of a vortex, α_{Re}, does depend on frequency:

$$\alpha_{Re} = \frac{Re(\varepsilon_{out}) - Re(\varepsilon_{in})}{Re(\varepsilon_{out}) + Re(\varepsilon_{in})} = \frac{\omega_{pl}^2 \left(\frac{2(T_C-T)}{T}\frac{\omega^2+\gamma^2}{\omega^2} - 2\frac{T-T_C}{T_C}\right)}{2(\omega^2+\gamma^2) - \omega_{pl}^2\left(\frac{2(T_C-T)}{T}\frac{\omega^2+\gamma^2}{\omega^2} - 2\frac{T}{T_C}\right)} \tag{16}$$

Clearly, as $(T_C - T)/T_C \to 0$, α_{Re} also vanishes. Thus the weak field approximation for $\varepsilon_{II}(x)$ given by Eq. (4) is valid at $(T_C - T)/T_C \to 0$, since both α_{Im} and α_{Re} vanish at temperatures close to T_C according to Eqs. (15) and (16).

To find an explicit expression for ω_0 we need to resolve Eq. (11) with respect to ω_0, taking into account the particular form of the dielectric function in Eqs. (13)-(14), using the following expression $\varepsilon_0(\omega) = (\varepsilon_{in}(\omega) + \varepsilon_{out}(\omega))/2$, and noting that ε_I is the dielectric constant region of region I, i.e., the region that the light is incident from, and which forms the interface with region II. Taking region I to be the vacuum, we set $\varepsilon_I = 1$. In this case Eq. (11) takes the form

$$\omega_0^4 + \omega_0^2\gamma^2 - \omega_p^2\omega_0^2 - \omega_p^2\gamma^2\frac{T_C - T}{T_C} = c^2 g^2 \left(2(\omega_0^2 + \gamma^2) - \omega_p^2 - \frac{\gamma^2\omega_p^2}{\omega_0^2}\frac{T_C - T}{T_C}\right) \tag{17}$$

III. DISCUSSION

Eq. (17) has two roots corresponding to the surface and bulk plasmon-polariton modes, respectively (the other roots correspond to imaginary frequencies). The root corresponding to the SPP mode is plotted in Fig. 1 (the solid curve) as a function of magnetic field. In the mechanism under our consideration only the SPPs are resonantly excited, while the frequency of the bulk polariton is out of the region of the analyzed resonance, and, thus, the bulk polariton mode is irrelevant. Using Eqs. (8)-(12), we can find ω_a and ω_b and calculate the width of the photonic band gap as a function of magnetic field under the assumption that $\alpha_{Re} \ll 1$ (which is, again, valid for T close to T_c). The results of these calculations are presented in Fig. 1 (dotted and dashed curves). The width of the band gap as a function of magnetic field strength is shown in Fig. 2. We can see that the size of the frequency band gap increases with increasing magnetic field.

In summary, the conditions which are necessary for the anomalous transmission of an electromagnetic wave through an array of vortices in a film of type-II superconductor have been analyzed. Under double-resonance conditions, resonant tunneling between surface plasmon polariton states at the two interfaces leads to enhancement of the transmission efficiency.[6] In the symmetric case, when the dielectric constants of the media surrounding the superconducting film coincide , i.e., $\varepsilon_I = \varepsilon_{III}$, the resonant enhancement of light transmission is achieved at all frequencies outside the SPP band gap. In the asymmetric structure of a film on a substrate ($\varepsilon_I \neq \varepsilon_{III}$), resonant enhancement of light transmission can be achieved only at specific discrete frequencies depending on ε_I, ε_{III}, and the period of the Abrikosov lattice $a(B,T)$, which is controlled by external magnetic field and temperature (see Eq. (1)).[6] By changing the external magnetic field and temperature one can control the period of the Abrikosov lattice, and hence the SPP band gap and the resonance frequency for the enhancement of the optical transmission in a film on a substrate. Near fields connected with the ends of the vortices (at opposite interfaces of the film) can be used for sensors. Two advantages of the proposed physical realization are the absence of the necessity of employing nanolithographic methods to produce the holes in the film and the possibility of controlling the optical properties of the film via external magnetic field and temperature.

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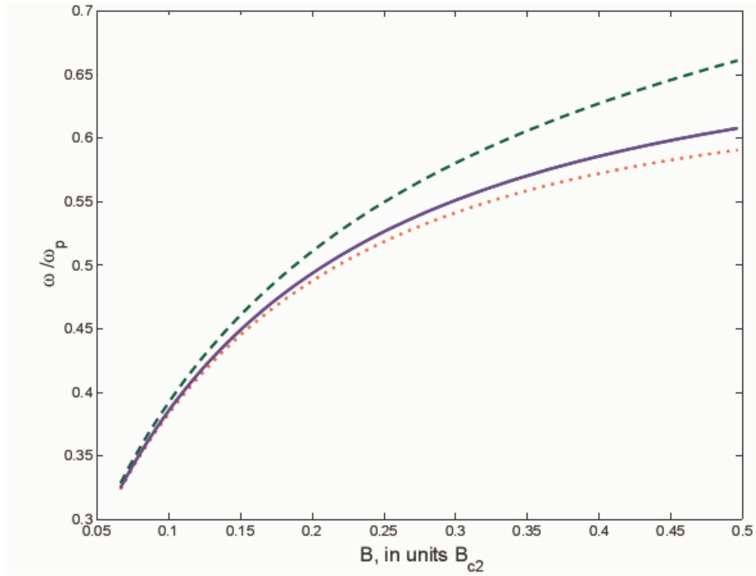

FIG. 1: Upper and lower frequencies of the first SPP band gap at $(T_C - T)/T_C = 0.022$. Solid curve corresponds to ω_0; dotted curve to ω_a; dashed curve to ω_b.

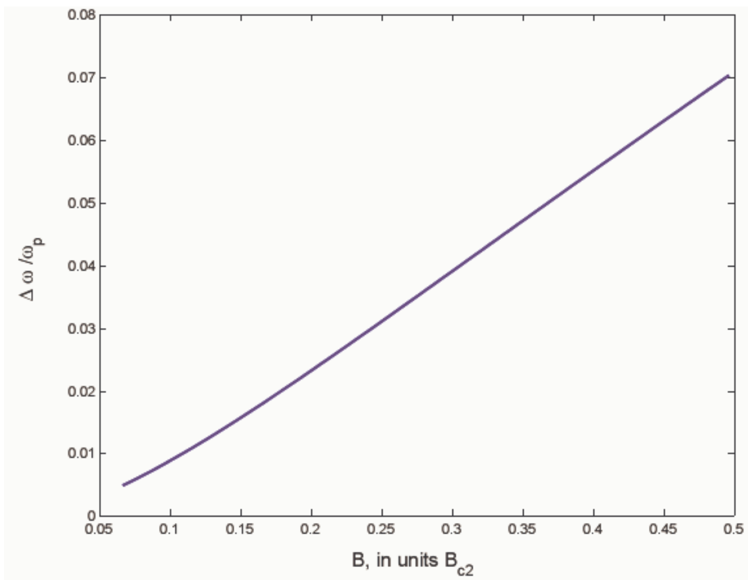

FIG. 2: The width of the SPP band gap at $(T_C - T)/T_C = 0.022$.